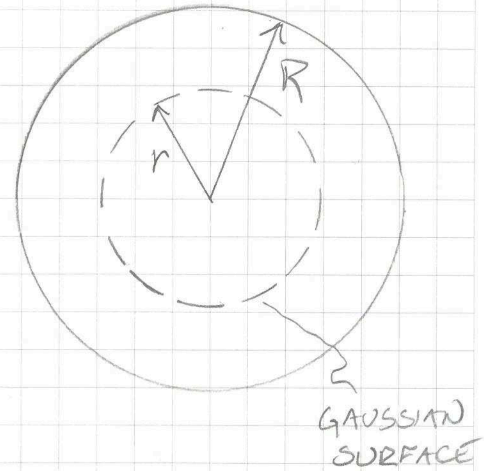
PROTON REPULSION WITHIN THE NUCLEUS



MODEL THE NUCLEUS AS A SPHERE OF UNIFORM CHARGE DENSITY, ρ. ALL THE OTHER PROTONS IN THE NUCLEUS GIVE A CHARGE OF $(Z-1)e$ IN WHICH EACH PROTON IS IMMERSED. THUS, FOR A RADIUS R

$$\rho = \frac{(Z-1)e}{\frac{4}{3}\pi R^3}$$

FOR A SPHERICAL GAUSSIAN SURFACE OF RADIUS r, GAUSS' LAW GIVES THE ELECTRIC FIELD AT THAT RADIUS

OUTSIDE R:

$$\oint \vec{E}\cdot d\vec{A} = 4\pi k Q_{INSIDE}$$

$$E(\cancel{4\pi} r^2) = \cancel{4\pi} k\,(Z-1)e$$

$$E_{r>R} = \frac{k(Z-1)e}{r^2} \qquad (1)$$

WITHIN R:

$$\oint \vec{E}\cdot d\vec{A} = 4\pi k Q_{INSIDE}$$

$$E(\cancel{4\pi r^2}) = \cancel{4\pi} k\left[\rho\left(\frac{4}{3}\pi \cancel{r^3}^{\,r}\right)\right]$$

$$= k\left[\frac{(Z-1)e}{\cancel{\frac{4}{3}\pi} R^3}\left(\cancel{\frac{4}{3}\pi} r\right)\right]$$

$$E_{r<R} = \frac{k(Z-1)e\,r}{R^3} \qquad (2)$$

TO DERIVE THE POTENTIAL ENERGY, FIND THE POTENTIAL DIFFERENCE BETWEEN TWO POINTS r_1 AND r_2 $\underbrace{(r_1 > r_2)}_{\text{BRINGING } p^+ \text{ IN}}$

$$V(r_2) - V(r_1) = -\int_{r_1}^{r_2} \vec{E}\cdot d\vec{r} = -\int_{r_1}^{r_2} \frac{k(Z-1)e}{R^3}\, r\,dr$$

$$= -\frac{k(Z-1)e}{R^3}\left(r_2^2 - r_1^2\right) = \frac{k(Z-1)e}{2R^3}\left(r_1^2 - r_2^2\right) \qquad (3)$$

FOR $r_1 = R$ AND $r_2 = r$, ANY INTERMEDIATE RADIUS

$$V(R) - V(r) = \frac{k(Z-1)e}{2R^3}\left(R^2 - r^2\right)$$

$\longrightarrow$

SO

$$V(r) = V(R) + \frac{k(z-1)e}{2R^3}(R^2 - r^2) \qquad (4)$$

WE CAN FIND $V(R)$ BY INTEGRATING $E_{r>R} = \frac{k(z-1)e}{r^2}$ FROM $r_1 = \infty$ TO $r_2 = R$

$$V(R) - V(\infty) = -\int_R^\infty \frac{k(z-1)e}{r^2} = -k(z-1)e\left(\frac{1}{\infty} - \frac{1}{R}\right)$$

DEFINING $V(\infty) = 0$ GIVES

$$V(R) = \frac{k(z-1)e}{R} \quad (r \geq R) \qquad (5)$$

SUBSTITUTING THIS BACK INTO (4) GIVES

$$V(r) = \frac{k(z-1)e}{R} + \frac{k(z-1)e}{2R^3}(R^2 - r^2)$$

OR

$$V(r) = \frac{k(z-1)e}{2R}\left(3 - \frac{r^2}{R^2}\right) \quad (r \leq R) \qquad (16.49)$$

FROM (5), (16.49) AND THE FACT THAT THE POTENTIAL DIFFERENCE TIMES THE CHARGE GIVES THE POTENTIAL ENERGY, WE HAVE

$$U_{COUL}(r) = eV(r) = \begin{cases} (z-1)\frac{ke^2}{r} & r \geq R \\ (z-1)\frac{ke^2}{2R}\left(3 - \frac{r^2}{R^2}\right) & r \leq R \end{cases}$$

FOR $r \to 0$

$$U(r=0) = (z-1)\frac{ke^2}{2R}(3-0) = \frac{3}{2}\frac{(z-1)ke^2}{R} \qquad (16.16)$$